Lecture 5 - Sep. 22

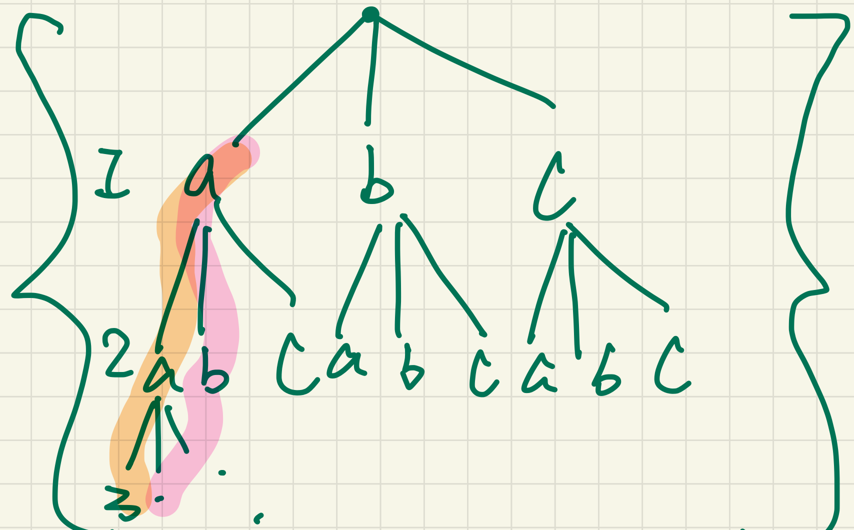
Lexical Analysis

RE: Exercises & Operator Precedence
DFA: Basics & Exercise

$$|\{a,b,\dots,z\}^5| = |\{a,b,\dots,z\}|^5$$

$$= 26^5$$

$$\boxed{\{a,b,c\}}^4 =$$



Alphabet

all strings of length 4

① $\Sigma_1 \subseteq \Sigma_2 \Rightarrow \Sigma_1^* \subseteq \Sigma_2^*$

Mathematical Induction

(Base Case) $\underbrace{\Sigma_1^0}_{\{\varepsilon\}} \subseteq \underbrace{\Sigma_2^0}_{\{\varepsilon\}}$ T.

(I.H.)

assume $\Sigma_1^n \subseteq \Sigma_2^n$ $(n>0)$.

(Prove) $\Sigma_1^{n+1} \subseteq \Sigma_2^{n+1}$

$c \in \Sigma_1$

to append to Σ_1^n,
it's guaranteed that
$c \in \Sigma_2$

②

$$\Sigma_1 \subseteq \Sigma_2 \Rightarrow \Sigma_1^* \subseteq \Sigma_2^*$$

$$\Sigma_1^*$$

$$= \Sigma_1^0 \cup \Sigma_1^1 \cup \Sigma_1^2 \cup \cdots$$

$$\subseteq \Sigma_2^0 \cup \Sigma_2^1 \cup \Sigma_2^2 \cup \cdots$$

$$= \Sigma_2^*$$

$L1$ { start with 0s — as many 1s as 0s }

$L2 = \{ xy \mid x \in \{0\}^* \wedge y \in \{1\}^+ \}$

$n \geq 0 \quad m \geq 0$

$L3 = \{ 0^n 1^m \mid m \geq n \}$

010 ✗

$$\boxed{\begin{array}{l} 001 \in L2 \\ 001 \notin L1 \end{array}}$$

$L1 \subset L2$

↳ not a string s

$s \in L1$

$s \notin L2$

$x \in \mathbb{N}$

$$L_2 = \left\{ a^x b^y c^z \;\middle|\; \begin{array}{l} x \geq 0 \wedge x \geq y + z \\ y \geq 1 \\ z \geq 1 \end{array} \right\}$$

$\underline{L_1} =$ ~~slide~~.

$ab \in$

$\underline{ab^1c^0} \in \frac{L_1}{\not\in L_2}$.

Exercise ✓

#s b's and c's at least as many as

#a's

Σ^* is $\boxed{\text{a language over } \Sigma}$

Σ^*

RE.

$$\boxed{\emptyset + L} = \emptyset \cup L = \underline{\underline{L}}.$$

$\Sigma^{\emptyset}$ vs L^{0}

$$\emptyset L = \{\underline{x}y \mid x \in \emptyset \wedge y \in L\} = \emptyset.$$

$$\emptyset^* = \emptyset^0 \cup \emptyset^1 \cup \underline{\underline{\emptyset^2}} \cup \cdots$$

$$= \{\varepsilon\} \cup \underbrace{\boxed{\{x \mid x \in \emptyset\}}}_{\emptyset} \cup \emptyset \cup \cdots$$

$$= \{\varepsilon\}$$

RE Specification: Exercise

Write a regular expression for the following language

$L1 = \{ w \mid w$ has alternating 0's and 1's $\}$

0 x

1 x

0 1 ✓

1 0 ✓

0 1 0 ✓

1 0 1 ✓

$L2 = (0\,(10)^+) + (1\,(01)^+)$

$01 \in L1$

$01 \notin L2$

RE: Operator Precedence

- Are RE_1 and RE_2 equivalent?
- A string in $L(RE_1)$ but <u>not</u> in $L(RE_2)$?
- A string in $L(RE_2)$ but <u>not</u> in $L(RE_1)$?

$\underbrace{10^*}_{L_1}$ vs. $\underbrace{(10)^*}_{L_2}$

$1(0^*)$

$$\boxed{\begin{array}{l} 1 \in L_1 \\ 1 \notin L_2 \end{array}} \qquad \boxed{\begin{array}{l} 1010 \notin L_1 \\ 1010 \in L_2 \end{array}}$$

$01^* + 1$ vs. $0(1^* + 1)$

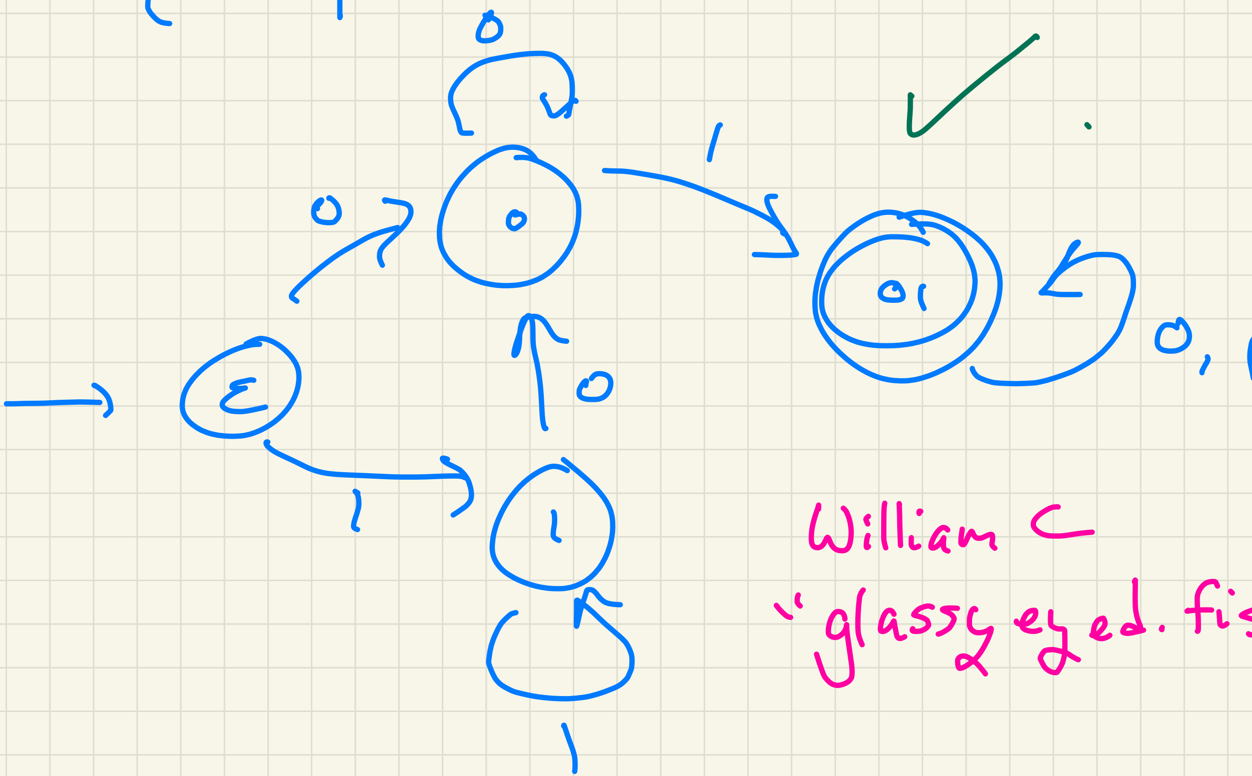
$0 + 1^*$ vs. $(0 + 1)^*$

DFA: Exercise

Draw the **transition diagram** of a **DFA** which **accepts/recognizes** the following language:

$\{ w \mid w \neq \varepsilon \wedge w \text{ has equal \# of alternating 0's and 1's} \}$

$\{ w \mid w \text{ contains } 01 \text{ as a substring} \}$



William C

"glassy eyed. fish"